

the precursor decays by an amount

$$(81) \quad du = dp_x / \rho_0 a_0 .$$

Under the assumed conditions, eq. (78) also applies along the path of the precursor. Combining eqs. (78) and (81) yields the relation

$$(82) \quad dp_x/dt = -F/2 .$$

The function F is expected in general to be quite complicated. We can get a qualitative picture of its effect by assuming the form, for compression only,

$$(83) \quad F = (p_x^e - p_x^s)/T, \quad p_x^e > p_x^s,$$

where $T = \text{constant}$. Compression by the precursor is assumed to be elastic, so p_x of eq. (82) lies on a metastable extension of the elastic compression curve, $p_x^e(V)$. Above the yield point there is a stress $p_x^s(V)$ which will finally be reached for the given volume V after a very long time. This is curve AB of Fig. 14 *b*). According to eqs. (82) and (83), decay of the precursor amplitude, $p_x \equiv p_x^e(V)$ continues until $p_x^e(V) = p_x^s(V)$, which occurs at the static value of the Hugoniot elastic limit. To see the effect more explicitly, note that

$$(84) \quad (d/dt)(p_x^e - p_x^s) = (1 - c^2/a^2)(dp_x^e/dt),$$

where $c^2 = K/\rho$, $a^2 = (K + 2\mu/3)/\rho$. If Poisson's ratio, ν , is independent of density, so is c^2/a^2 . Then eqs. (82)-(84) can be integrated to yield

$$(85) \quad p_x^e(V) - p_x^s(V) = (p_x^e - p_x^s)_0 \exp[-x/x_0],$$

where

$$(86) \quad x_0 = 2TD/(1 - c^2/a^2) .$$

Integrating eq. (84) under the assumption that $\nu = \text{constant}$ enables us to simplify eq. (85):

$$(87) \quad p_x^e - p_{\text{HEL}}^s = (p_x^e - p_{\text{HEL}}^s)_0 \exp[-x/x_0],$$

where p_{HEL}^s is the static value of the Hugoniot elastic limit, related to the static yield strength by eq. (47).

Equation (82) was derived on the assumptions that the precursor follows a characteristic and that the energy equation, eq. (3), does not affect the prop-

agation process. A more rigorous expression can be obtained by combining eq. (77) with eqs. (1)-(3) and specializing the result along the shock path [8]:

$$(88) \quad \frac{Dp_x}{Dx} = \left(1 - \frac{u}{D}\right) \frac{(D-u)^2 - a^2}{\frac{3}{2}(D-u)^2 + a^2/2} \frac{\partial p_x}{\partial x} - \frac{(D-u)^2}{D} \frac{F'}{\frac{3}{2}(D-u)^2 + a^2/2},$$

$$(89) \quad F' = (1 - \alpha\Gamma\gamma/2\mu)F.$$

Here the block derivative, D/Dx , refers to differentiation along the shock path, $\partial p_x/\partial x$ is evaluated immediately behind the precursor front, and F' is a modification to F resulting from the assumption that a fraction α of plastic work goes into heat. In eq. (89), Γ is the Gruneisen parameter. F' and F differ by less than 10% for metals in which plastic flow occurs.

Under the assumptions that $D-u=a$ and $\alpha=0$, eq. (88) reduces to eq. (82).

Considerable effort in recent years has been devoted to attempts to relate the relaxation function F of eq. (75) to the motion and multiplication of dislocations. The basic relation is

$$(90) \quad dE^p/dt = hNbv = F/2\mu,$$

where N is the number of dislocations per unit area, b is the Burgers vector, h is a numerical constant the order of units, and v is the mean velocity of dislocations. Since $E_p = 2\varepsilon_1/3$ in uniaxial strain, eq. (90) becomes

$$(91) \quad d\varepsilon_1/dt = 3hNbv/2.$$

There are various models for multiplication and motion of dislocations. One which is frequently used is due to GILMAN:

$$(92) \quad N = N_{om}(1 + Ae^p),$$

$$(93) \quad v = v_{max} \exp[-D/\tau],$$

where

N_{om} = initial density of mobile dislocations,

v_{max} = maximum dislocation velocity $\sim v_{shear}$,

D = drag coefficient,

A = multiplication coefficient,

τ = resolved shear stress = $(p_x - p_y)/2$.